

History of Binomial Theory

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Abstract— This article shows the history of binomial theory. From Euclid II to Sir Issac Newton, it discusses every individual ever involved with this mathematical formula and method. Besides, it focuses on how it transformed from one form to another and who did exactly what in its development as a theory. It also sheds some light on the contemporary standing of the theory.



1 Euclid II

The binomial theorem is a very important theory in mathematics and has always played massive role in the development of mathematics, "both in algebra and analysis in 4th century B.C." (Goss).

Euclid II's (325 BC - 265BC) binomial expansion using geometry is earliest example and trace of this theory that has been developed as until now. "According to his explanation, the area of a square is equal to the sum of areas of rectangles contained within it" (Coolidge).

$$a^2 + b^2 + 2ab = (a + b)^2$$

2 Chu-Shih-Cheih

Chu-Shih-Cheih, from china is a mathematician, to whom the following diagram is referred to.

```

1
1 1
1 2 1
1 3 3 1
. . . . .
1 8 . . . . . 8 1
    
```

The horizontal arrangement indicates its derivation from binomial expansion, however there is nothing to prove that. (Goss)

3 Michael Stifel

After this Chinese mathematician, the name of Michael Stifel is taken a lot for having developed a similar diagram. "This diagram was published in Arithmetica Integra in 1544" (Coolidge).

```

1
2
3 3
4 6
5 10 10
6 15 20
7 21 35 35
8 28 56 70
    
```

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9 36 84 126 126
10 45 120 210 252
    
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Stifel was very much into approximation extraction of roots and this table supports that idea. In the first column, there are the 1st ten integers. In each following columns, the column starts with two values lower than the preceeding one. It starts with the number immediate to its left and each following number in the column is sum of number right above and left of it. So the general formula would be,

$$(a + b)^n = (a + b)^{n-1} (a + b)$$

4 Blaise Pascal

In 3rd century B.C. Indian mathematician Pingala proposed the pascal's triangle, i.e. presenting binomials coefficients in a triangle. Blaise Pascal (1632 - 1662) was a French mathematician who is credited for having discovered the pattern in binomial triangle. (Goss)

The diagram for that triangle is given below

```

1 1 1 1 1 1
1 2 3 4 5 6
1 3 6 10 15
1 4 10 20
1 5 15
1 6
1
    
```

This diagram proposed by pascal was based on the same rule that Stifel's diagram based on. The general form of the pascal's triangle can be written like below.

$$(n r) = n(n-1)(n-2) \dots (n-r+1) / r(r-1)(r-2) \dots 1$$

5 Al-Karaji

Around 10th century, Indian mathematician and Persian mathematician, Halayudha and al-Karaji proposed and derived similar formula and diagram as the Chinese mathematician did before B.C. Al-karaji (953 A.D - 1029 A.D) was an engineer and a mathematician from Persia. He introduced binomial expansion in algebra. Using 'mathematical induction', he proved binomial expansion. He also created a table of coefficients that was later pronounced as binomial triangle. (Coolidge)

6 Omar Khayyam

After Al-Karaji, Omar Khayyam (1048 – 1113) generalized binomial expansion. He was a Persian mathematician, philosopher, poet and astronomer. He also found the n th root based on binomial expansion and coefficient. Other than him, Zhu Shijie (1260 – 1320) is also credited to have published the earliest binomial triangles in 1303. He was from southern China and a mathematician. ("The History Of The Binomial Theorem")

3 Gregory and Newton

In 1670, James Gregory gave the formula for the binomial expansion of a fractional power.

The formula he gave was

$$\log b + a/c [\log (b + d) - \log b] = e + a$$

In 1665, Sir Issac Newton's contribution to binomial expansion was discovered, however it was also discussed in a letter to Oldenburf in 1676. Sir Issac Newton (1642 – 1727) developed formula for binomial theorem that could work for negative and fractional numbers using calculus. Impressed by John Wallis work on calculating the area under the curve, Newton proposed the expansion of $(1 - x^2)^s$. He simply replaced 'n' with 's' from John's formula. He calculated the Maclaurin series for $(1 - x^2)^{1/2}$, $(1 - x^2)^{3/2}$, and $(1 - x^2)^{1/3}$. (Goss)

Newton wanted to find the areas under the curves of the above formula or equations.

REFERENCES

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